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REGENERATIVE SIMULATION METHODS FOR LOCAL AREA COMPUTER 1/1

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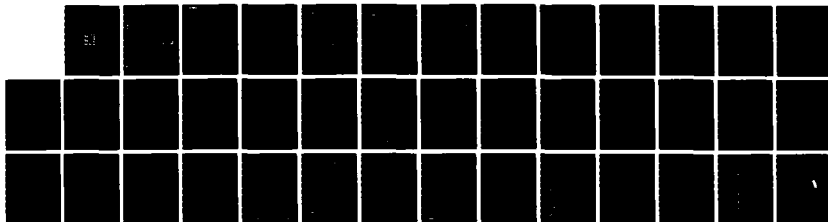
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ABSTRACT: Local area computer network simulations are inherently non-Markovian in that the underlying stochastic process cannot be modeled as a Markov chain with countable state space. We restrict attention to local network simulations with an underlying stochastic process that can be represented as a generalized semi-Markov process (GSMP). Using "new better than used" distributional assumptions and sample path properties of the GSMP, we provide a "geometric trials" criterion for recurrence in this setting. We also provide conditions which ensure that a GSMP is a regenerative process and that the expected time between regeneration points is finite. Steady-state estimation procedures for ring and bus network simulations follow from these results.



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**REGENERATIVE SIMULATION METHODS
FOR LOCAL AREA COMPUTER NETWORKS**

by

Peter J. Haas and Gerald S. Shedler

TECHNICAL REPORT NO. 1

June 1984

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1. INTRODUCTION

It is difficult to establish estimation procedures for local area computer network simulations that explicitly incorporate access control algorithms. Such simulations (see e.g., Iglehart and Shedler [8,9], Loucks, Hamacher, and Probas [12]) are inherently non-Markovian in the sense that the underlying stochastic process cannot be modeled as a Markov chain with countable state space. Following [8] we restrict attention to local network simulations whose underlying stochastic process can be represented as a generalized semi-Markov process (GSM).

Although steady-state estimation for an arbitrary GSM is a formidable problem, estimation procedures [8,9] are available for GSM's that are regenerative processes. To establish the regenerative property for a GSM, it is necessary to show the existence of an infinite sequence of random times at which the process probabilistically restarts. It is often the case that a GSM associated with a local area network simulation probabilistically restarts when a particular event triggers a transition to some fixed state. For specific models, however, it is nontrivial to determine conditions (distributional assumptions) under which the underlying GSM returns infinitely often to the fixed state.

A "geometric trials" argument given in [8] establishes a recurrence criterion for a stochastic process $\{X(t): t \geq 0\}$ with right-continuous and piecewise constant sample paths and countable state space, S . Let $\{T_n: n \geq 0\}$ be an increasing sequence of finite state transition times for $\{X(t): t \geq 0\}$. The process $\{X(T_n): n \geq 0\}$ hits state $s' \in S$ infinitely often with probability one provided that $P\{X(T_n) = s' | X(T_{n-1}), \dots, X(T_0)\} \geq \delta$ a.s. for some $\delta > 0$. This geometric trials recurrence criterion avoids the often unrealistic "positive density" assumptions needed in arguments (cf., [7]) based on general state space Markov chain theory.

In this paper, using sample path properties of the GSP, we provide conditions which permit application of the geometric trials recurrence criterion in the GSP setting. Our approach is to postulate the existence of a distinguished random time in the interval $[T_{n-1}, T_n]$ and a set of distinguished events determined by the state of the system at the distinguished time such that $X(T_n) = s'$ if each of the distinguished events occurs "soon enough" before time T_n . We show that $\{X(T_n): n \geq 0\}$ hits state s' infinitely often with probability one if the clock setting distributions associated with the distinguished events have "new better than used" distributions and satisfy a "positivity" condition. We also provide additional conditions on the building blocks of the GSP which ensure that the successive times at which $\{X(T_n): n \geq 0\}$ hits state s' are regeneration points for the process $\{X(t): t \geq 0\}$ and that the expected time between regeneration points is finite.

2. REGENERATIVE GENERALIZED SEMI-MARKOV PROCESSES

Heuristically, a GSP (Matthes [13], König, Matthes and Nawrotzki [10], [11]) moves from state to state in accordance with the occurrence of events associated with the occupied state. Each of the several possible events associated with a state compete to trigger the next transition and each of these events has its own distribution for determining the next state. At each state transition of the GSP, new events may be scheduled. For each of these new events, a clock indicating the time until the event is scheduled to occur is set according to an independent (stochastic) mechanism. If a scheduled event does not trigger a transition but is associated with the next state, its clock continues to run; if such an event is not associated with the next state, it is abandoned.

Following Whitt [16], formal definition of a GSP is in terms of a general state space Markov chain (GSPC) which describes the process at successive epochs of state

transition. Let S be a finite or countable set of states and $E = \{e_1, e_2, \dots, e_M\}$ be a finite set of events. For $s \in S$, $E(s)$ denotes the set of all events that can occur when the osmr is in state s . When the process is in state s , the occurrence of an event $e \in E(s)$ triggers a transition to a state s' . We denote by $p(s'; s, e)$ the probability that the new state is s' given that event e triggers a transition in state s . For each $s \in S$ and $e \in E(s)$ we assume that $p(\cdot; s, e)$ is a probability mass function. The actual event $e \in E(s)$ which triggers a transition in state s depends on clocks associated with the events in $E(s)$ and the speeds at which these clocks run. Each such clock records the remaining time until the event triggers a state transition. We denote by r_{si} (≥ 0) the deterministic rate at which the clock c_i associated with event e_i runs in state s ; for each $s \in S$, $r_{si} = 0$ if $e_i \notin E(s)$. We assume that $r_{si} > 0$ for some $e_i \in E(s)$. (Typically in applications, all speeds r_{si} are equal to one. There are, however, models in which speeds other than unity as well as state-dependent speeds are convenient. For example, zero speeds are needed in queuing systems with service interruptions of the preemptive-resume type; cf. Shedler and Southard [14].)

For $s \in S$ define

$$C(s) = \{(c_1, \dots, c_M) : c_i \geq 0 \text{ and } c_i > 0 \text{ if and only if } e_i \in E(s);$$

$$(2.1) \quad c_i r_{si}^{-1} \neq c_j r_{sj}^{-1} \text{ for } i \neq j \text{ with } c_i c_j r_{si} r_{sj} > 0\}.$$

The conditions in Equation (2.1) ensure that no two events simultaneously trigger a transition (as defined below). The set $C(s)$ is the set of possible clock readings in state s . The clock c_i and event e_i are said to be active in state s if $e_i \in E(s)$. For $s \in S$ and $c \in C(s)$,

let

$$(2.2) \quad \dot{t} = \dot{t}(s, c) = \min_{\{i: e_i \in E(s)\}} \{c_i r_{si}^{-1}\},$$

where $c_i r_{si}^{-1}$ is taken to be $+\infty$ when $r_{si} = 0$. Also set

$$(2.3) \quad \dot{c}_i = \dot{c}_i(s, c) = c_i - \dot{t}(s, c) r_{si}, \quad e_i \in E(s)$$

and

$$(2.4) \quad \dot{i} = \dot{i}(s, c) = i \text{ such that } e_i \in E(s) \text{ and } \dot{c}_i(s, c) = 0.$$

Beginning in state s with clock vector c , $\dot{t}(s, c)$ is the time to the next state transition and $\dot{i}(s, c)$ is the index of the unique triggering event $e^* = e^*(s, c) = e_{\dot{i}(s, c)}$.

At a transition from state s to state s' triggered by event e^* , new clock times are generated for each $e' \in N(s'; s, e^*) = E(s') - (E(s) - \{e^*\})$. The distribution function of such a new clock time is denoted by $F(\cdot; s', e', s, e^*)$ and we assume that $F(0; s', e', s, e^*) = 0$. For $e' \in O(s'; s, e^*) = E(s') \cap (E(s) - \{e^*\})$, the old clock reading is kept after the transition. For $e' \in (E(s) - \{e^*\}) - E(s')$, event e' ceases to be scheduled after the transition.

Next consider a GSSMC $\{(S_n, C_n): n \geq 0\}$ having state space

$$\Sigma = \bigcup_{s \in S} (\{s\} \times C(s))$$

and representing the state (S_n) and vector (C_n) of clock readings at successive state transition epochs. (The n th coordinate of the vector C_n is denoted by $C_{n,i}$.) The transition kernel of the Markov chain $\{(S_n, C_n): n \geq 0\}$ is

$$(2.5) \quad P((s, c), A) = P(s'; s, e^*) \prod_{e_i \in N(s')} F(a_i; s', e_i, s, e^*) \prod_{e_i \in O(s')} 1_{[0, a_i]}(c_i),$$

where $N(s') = N(s'; s, s^*)$, $O(s') = O(s'; s, s^*)$, and

$$A = \{s'\} \times \{(c'_1, \dots, c'_M) \in C(s') : c'_i \leq a_i \text{ for } e_i \in E(s')\}.$$

The set A is the subset of Σ which corresponds to the GSR entering state s' with the reading c_i on the clock associated with event $e_i \in E(s')$ set to a value in $[0, a_i]$.

Finally, the GSR is a piecewise constant continuous time process constructed from the GSRAC $\{(S_n, C_n) : n \geq 0\}$ in the following manner. Set $t_0 = 0$ and denote by t_n the time of the n th state transition, $n \geq 0$. (We assume that

$$P\{\sup_{n \geq 1} t_n = +\infty | (S_0, C_0)\} = 1 \text{ a.s.}$$

for all initial states (S_0, C_0) .) Then set

$$(2.6) \quad X(t) = S_{N(t)},$$

where

$$N(t) = \max \{n \geq 0 : t_n \leq t\}.$$

The process $\{X(t) : t \geq 0\}$ defined by Equation (2.6) is a GSR. We assume from now on that all speeds r_{ij} are equal to 1.

The characteristic property of a regenerative stochastic process (Smith [15]) is that there exist random time points, referred to as *regeneration points* or *regeneration times*, at which the process probabilistically restarts. The essence of regeneration is that the evolution of the process in a cycle (i.e., between any two successive regeneration points) is a probabilistic replica of the process in any other cycle. In the presence of mild regularity conditions, a regenerative stochastic process $\{X(t) : t \geq 0\}$ has a limiting distribution $(X(t) \rightarrow X \text{ as } t \rightarrow \infty)$ provided that the expected time between regeneration

points is finite. Furthermore, the regenerative structure ensures that the behavior of the process in a cycle determines the expected value of a function of the limiting random variable X as a ratio of expected values. These results have important implications for simulation and are the basis for the regenerative method for simulation analysis; see Crane and Iglehart [3].

(2.7) DEFINITION. The real (possibly vector-valued) stochastic process $\{X(t): t \geq 0\}$ is a *regenerative process in continuous time* provided that:

- (i) there exists a sequence of stopping times $\{T_k: k \geq 0\}$ such that $\{T_{k+1} - T_k: k \geq 0\}$ are independent and identically distributed;
- (ii) for every sequence of times $0 < t_1 < t_2 < \dots < t_m$ ($m \geq 1$) and $k \geq 0$, the random vectors $\{X(t_1), \dots, X(t_m)\}$ and $\{X(T_k + t_1), \dots, X(T_k + t_m)\}$ have the same distribution and the processes $\{X(t): t < T_k\}$ and $\{X(T_k + t): t \geq 0\}$ are independent.

Recurrence properties of the underlying stochastic process of a discrete-event simulation are needed to establish estimation procedures based on regenerative processes. Lemma (2.8) is a special case of a generalized Borel-Cantelli lemma due to Doob [4, p. 324]; see [8, Lemma 4] for an elementary proof using a "geometric trials" argument.

(2.8) LEMMA. Let $\{Y_n: n \geq 0\}$ be a sequence of random variables defined on a probability space (Ω, \mathcal{F}, P) and taking on values in a set, S . Let $s' \in S$. Suppose that there exists $\delta > 0$ such that

$$P\{Y_n = s' \mid Y_{n-1}, \dots, Y_0\} \geq \delta \text{ a.s.}$$

for all $n \geq 1$. Then $P\{Y_n = s' \text{ i.o.}\} = 1$.

Lemma (2.8) provides a means of showing that the underlying stochastic process of a simulation returns infinitely often to a fixed state. Specifically, let $\{X(t): t \geq 0\}$ be a stochastic process with right-continuous and piecewise constant sample paths and countable state space, S . Let $s' \in S$ and suppose that $\{T_n: n \geq 0\}$ is an increasing sequence of finite $(T_n < \infty \text{ a.s.})$ state transition times for $\{X(t): t \geq 0\}$ such that

$$P\{X(T_n) = s' \mid X(T_{n-1}), \dots, X(T_0)\} \geq \delta \text{ a.s.}$$

for some $\delta > 0$. Then $P\{X(T_n) = s' \text{ i.o.}\} = 1$ by Lemma (2.8) (with $Y_n = X(T_n)$).

Using "new better than used" distributional assumptions and the sample path structure of the process, Proposition (2.11) provides sufficient conditions for recurrence in the Gsmr setting.

(2.9) DEFINITION. The distribution F of a positive random variable A is *new better than used (NBU)* if

$$P\{A > x + y \mid A > y\} \leq P\{A > x\}$$

for all $x, y \geq 0$.

See Barlow and Proschan [1] for a discussion of NBU distributions. Note that every increasing failure rate (IFR) distribution is NBU. Also, if A and B are independent random variables with NBU distributions, then the distributions of $A + B$, $\min(A, B)$, and $\max(A, B)$ are NBU.

Let $\{X(t): t \geq 0\}$ be a Gsmr with finite state space, S , and event set, E . Suppose that $\{T_n: n \geq 0\}$ is an increasing sequence of finite $(T_n < \infty \text{ a.s.})$ state transition times such that for some $e' \in E$ and $s' \in S$: $T_0 = 0$ and

$$(2.10) \quad T_n = \inf\{t > T_{n-1}: \text{at time } t \text{ event } e^+ \text{ triggers a transition in some state } s^+ \in S^+\},$$

$n \geq 1$.

Let $s_0^+ \in S$. Proposition (2.11) postulates the existence of a distinguished random time (T_n^+) in the interval $[T_{n-1}, T_n)$ defined by Equation (2.10), and a set $(E^+(s_n^+))$ of distinguished events determined by the state s_n^+ of the system at time T_n^+ such that $X(T_n) = s_0^+$ when each of the distinguished events $e_k^+(s_n^+)$ occurs prior to some time $T_n^+ + R_{n,k}(s_n^+) (> T_n^+)$. The proposition asserts that $\{X(T_n): n \geq 0\}$ hits state s_0^+ infinitely often with probability one if the clock setting distributions associated with the distinguished events are NBU and satisfy a "positivity" condition which guarantees the existence of $\delta > 0$ as in Lemma (2.8).

Let $\{T_n^+: n \geq 0\}$ be a sequence of state transition times and denote the state space of $\{X(T_n^+): n \geq 0\}$ by S^+ . For $s^+ \in S^+$, let

$$E^+(s^+) = \{e_1^+(s^+), \dots, e_{k(s^+)}^+(s^+)\} \subseteq E(s^+)$$

and set

$$E^+ = \bigcup_{s^+ \in S^+} E^+(s^+).$$

When $X(T_n^+) = s^+$ we denote by $S_{n,k}(s^+)$ the latest time less than or equal to T_n^+ at which the clock associated with event $e_k^+(s^+)$ was set, and by $A_{n,k}(s^+)$ the setting on the clock at time $S_{n,k}(s^+)$.

(2.11) PROPOSITION. Assume that there exist state transition times $\{T_n^+ : n \geq 0\}$ and for $s^+ \in S^+$ event sets $E^+(s^+)$ and identically distributed strictly positive random vectors $(R_{n,1}(s^+), \dots, R_{n,k(s^+)}(s^+))$, independent of $\{A_{n,1}(X(T_n^+)), \dots, A_{n,k(s^+)}(X(T_n^+))\}$ and $\{X(t) : 0 \leq t \leq T_n^+\}$, such that:

(i) $T_{n-1} \leq T_n^+$ a.s. and for $x_0, x_1, \dots, x_{n-1} \in S$ and $s^+ \in S^+$,

$$\begin{aligned} P\{X(T_n) = s_0^+, X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\} \\ \geq P\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+); \\ X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}; \end{aligned}$$

(ii) for all $e^+ \in E^+$, the clock setting distribution $F(\cdot; s', e^+, s, e) = F(\cdot; e^+)$ and is NBU;

(iii) there exists $\delta > 0$ such that for $s^+ \in S^+$

$$\delta(s^+) = P\{A_k(s^+) \leq R_{n,k}(s^+), k = 1, 2, \dots, k(s^+)\} \geq \delta,$$

where the random variable $A_k(s^+)$ has distribution $F(\cdot; e_k^+(s^+))$ and $\{A_1(s^+), \dots, A_{k(s^+)}(s^+)\}$ are mutually independent and independent of $(R_{n,1}(s^+), \dots, R_{n,k(s^+)}(s^+))$.

Then

$$P\{X(T_n) = s_0^+ | X(T_{n-1}), \dots, X(T_0)\} \geq \delta \text{ a.s.}$$

so that $P\{X(T_n) = s_0^+ \text{ i.o.}\} = 1$.

Proof: Let $s^+ \in S^+$ and $x_0, \dots, x_{n-1} \in S$. Lemma (A.6) of the Appendix shows that

$$P\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+);$$

$$X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}.$$

$$(2.12) \quad \geq \delta P\{X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}.$$

Using Equation (2.12),

$$P\{X(T_n) = s_0^i, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}$$

$$= \sum_{s^+ \in S^+} P\{X(T_n) = s_0^i, X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}$$

$$\geq \sum_{s^+ \in S^+} P\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+);$$

$$X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\},$$

$$\geq \sum_{s^+ \in S^+} \delta P\{X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}$$

$$= \delta P\{X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}.$$

It follows that

$$P\{X(T_n) = s_0^i | X(T_{n-1}), \dots, X(T_0)\} \geq \delta \text{ a.s.}$$

and Lemma (2.8) implies that $P\{X(T_n) = s_0^i \text{ i.o.}\} = 1$. \square

Proposition (2.13) gives a set of conditions on the building blocks of a csmp which ensure that the process is regenerative and that the expected time between regeneration points is finite.

(2.13) PROPOSITION. Let $\{T_n; n \geq 0\}$ be an increasing sequence of stopping times that are finite ($T_n < \infty$ a.s.) state transition times as in Equation (2.10). Suppose that there exists $s_0' \in S$ and $\delta > 0$ such that

$$(2.14) \quad P\{X(T_n) = s_0' | X(T_{n-1}), \dots, X(T_0)\} \geq \delta \text{ a.s.}$$

Also suppose that for $s' \in S'$, (i) the set $O(s_0'; s', e') = E(s_0') \cap (E(s') - \{e'\}) = \emptyset$, (ii) the set $N(s_0'; s', e') = N(s_0'; s, e')$, and (iii) the clock setting distribution $F(\cdot; s_0', e', s', e') = F(\cdot; s_0', e', s, e')$ for all $e' \in N(s_0'; s', e')$. Then $\{X(t); t \geq 0\}$ is a regenerative process in continuous time. Moreover, if

$$E\{T_{n+1} - T_n\} \leq c < \infty$$

for all $n \geq 0$ then the expected time between regeneration points is finite.

Proof: Using Lemma (2.8), Equation (2.14) implies that event e' triggers a transition to state s_0' from some state $s' \in S'$ infinitely often with probability one. Furthermore, at such a time T_n , the only clocks that are active have just been set since $O(s_0'; s', e') = \emptyset$ for all $s' \in S'$. The joint distribution of $X(T_n)$ and the clocks set at time T_n depends on the past history of $\{X(t); t \geq 0\}$ only through s_0' , the previous state s' , and the trigger event e' . Since the new events and clock setting distributions are the same for all s' , the process $\{X(t); t \geq 0\}$ probabilistically restarts whenever $\{X(T_n); n \geq 0\}$ hits state s_0' .

To show that the expected time between regeneration points is finite, assume for convenience that $X(T_0) = X(0) = s_0'$. Set $X_n = X(T_n)$ and $D_n = T_{n+1} - T_n$, $n \geq 0$. Observe

that the random indices β_n such that $X_{\beta_n} = X(T_{\beta_n}) = s'_0$ form a sequence of regeneration points for the process $\{(X_n, D_n): n \geq 0\}$; this follows from the fact that the process $\{D_n: n \geq 1\}$ starts from scratch when $X(T_{\beta_n}) = s'_0$. Let $\tau_k = \beta_{k+1} - \beta_k$, $k \geq 1$. The τ_k are i.i.d. as τ_1 and the argument in the proof of Lemma 4 in [8] shows that

$$P\{\tau_1 > n\} \leq (1 - \delta)^n$$

so that $E\{\tau_1\} < \infty$. Thus the expected time between regeneration points for the process $\{(X_n, D_n): n \geq 0\}$ is finite. Since $E\{\tau_1\} < \infty$ and Equation (2.14) ensures that τ_1 is aperiodic, $(X_n, D_n) \Rightarrow (X, D)$ as $n \rightarrow \infty$. Using the continuous mapping theorem we have $D_n \Rightarrow D$ as $n \rightarrow \infty$ and, since $D_n \geq 0$ and $E\{D_n\} \leq c < \infty$,

$$E\{|D|\} = E\{D\} \leq \lim_{n \rightarrow \infty} E\{D_n\} \leq c < \infty$$

by Theorem 25.11 in [2]. Since τ_1 is aperiodic, $E\{\tau_1\} < \infty$, and $E\{|D|\} < \infty$,

$$E\{D\} = \frac{E\left\{\sum_{j=0}^{\tau_1-1} D_n\right\}}{E\{\tau_1\}}$$

so that

$$E\left\{\sum_{j=0}^{\tau_1-1} D_n\right\} < \infty$$

and the expected time between regeneration points for $\{X(s): s \geq 0\}$ is finite. \square

Note that the state transition times $\{T_n: n \geq 0\}$ defined by Equation (2.10) are necessarily stopping times if for all $s', s'' \in S$

$$(2.15) \quad e = e' \text{ whenever } p(s''; s', e) > 0 \text{ and } p(s''; s', e') > 0.$$

3. RING AND BUS NETWORK MODELS

The following examples illustrate the use of the GSNP model as a formal specification of a discrete-event simulation of a local area computer network and the application of Propositions (2.11) and (2.13). These results are also applicable to the token ring and collision-free bus network models in Examples (2.7) and (2.9) of [9].

(3.1) **EXAMPLE (Token ring).** Consider a unidirectional ring network having a fixed number of *ports*, labelled $1, 2, \dots, N$ in the direction of signal propagation; see Figure 1. At each port *message packets* arrive according to a random process and queue externally. A single *control token* (denoted by T in Figure 1) circulates around the ring from one port to the next. The time for the token to propagate from port N to port 1 is a positive constant, R_N , and the time for the token to propagate from port $j-1$ to port j is a positive constant, R_{j-1} , $j = 2, 3, \dots, N$. When a port observes the token and there is a packet queued for transmission, the port converts the token to a *connector* (C) and transmits a packet followed by the token pattern; the token continues to propagate if there is no packet queued for transmission. By destroying the connector prefix the port removes the transmitted packet when it returns around the ring. Assume that the time for port j to transmit a packet is a positive random variable, L_j , with finite mean. Also assume that packets arrive at individual ports randomly and independently of each other; i.e., the time from end of transmission by port j until the arrival of the next packet for transmission by port j is a positive random variable, A_j , with finite mean. Note that there is at most one packet queued for transmission at any time at any particular port.

Set

$$(3.2) \quad X(i) = (Z_1(i), \dots, Z_N(i); M(i); N(i)),$$

where

$$Z_j(t) = \begin{cases} 1 & \text{if there is a packet queued for transmission at port } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$M(t) = \begin{cases} j & \text{if port } j \text{ is transmitting a packet at time } t \\ 0 & \text{if no port is transmitting a packet at time } t \end{cases}$$

$N(t) = 1$ if at time t port N is transmitting a packet or the token is propagating to port 1, and $N(t) = j$ if at time t port $j-1$ is transmitting a packet or the token is propagating to port j , $j = 2, \dots, N$.

The process $\{X(t); t \geq 0\}$ defined by Equation (3.2) is a GSP with a finite state space, S , and event set, $E = \{e_1, \dots, e_{N+2}\}$, where e_{N+2} = "observation of token," e_{N+1} = "end of transmission," and e_j = "arrival of packet for transmission by port j ," $j = 1, 2, \dots, N$. For $s = (z_1, \dots, z_N; m; n) \in S$, the event sets $E(s)$ are as follows. The event "end of transmission" $\in E(s)$ if and only if $m > 0$ and "observation of token" $\in E(s)$ if and only if $m = 0$. The event "arrival of packet for transmission by port j " $\in E(s)$ if and only if $z_j = 0$ and $m \neq j$.

If e = "end of transmission," then the state transition probability $p(s'; s, e) = 1$ when

$$s = (z_1, \dots, z_N; m; m+1) \in S \text{ with } 0 < m < N \text{ and } s' = (z_1, \dots, z_N; 0; m+1)$$

and when

$$s = (z_1, \dots, z_N; N; 1) \in S \text{ and } s' = (z_1, \dots, z_N; 0; 1).$$

If e = "observation of token," then $p(s'; s, e) = 1$ when

$$s = (z_1, \dots, z_{n-1}, 1, z_{n+1}, \dots, z_N; 0; n) \in S \text{ with } n < N \text{ and } s' = (z_1, \dots, z_{n-1}, 0, z_{n+1}, \dots, z_N; n; n+1),$$

when

$$s = (z_1, \dots, z_{N-1}, 1; 0; N) \in S \text{ and } s' = (z_1, \dots, z_{N-1}, 0; N; 1),$$

when

$$s = (z_1, \dots, z_{n-1}, 0, z_{n+1}, \dots, z_N; 0; n) \in S \text{ with } n < N \text{ and } s' = (z_1, \dots, z_{n-1}, 0, z_{n+1}, \dots, z_N; 0; n+1),$$

and when

$$s = (z_1, \dots, z_{N-1}, 0; 0; N) \text{ and } s' = (z_1, \dots, z_{N-1}, 0; 0; 1).$$

If e = "arrival of packet for transmission by port j ," then $p(s'; s, e) = 1$ when

$$s = (z_1, \dots, z_{j-1}, 0, z_{j+1}, \dots, z_N; m; m+1) \in S \text{ with } m \neq j \text{ and } 0 < m < N$$

and

$$s' = (z_1, \dots, z_{j-1}, 1, z_{j+1}, \dots, z_N; m; m+1),$$

when

$$s = (z_1, \dots, z_{j-1}, 0, z_{j+1}, \dots, z_N; N; 1) \in S \text{ with } N \neq j \text{ and } s' = (z_1, \dots, z_{j-1}, 1, z_{j+1}, \dots, z_N; N; 1),$$

and when

$$s = (z_1, \dots, z_{j-1}, 0, z_{j+1}, \dots, z_N; 0; n) \in S \text{ and } s' = (z_1, \dots, z_{j-1}, 1, z_{j+1}, \dots, z_N; 0; n).$$

All other state transition probabilities $p(s'; s, e)$ are equal to zero.

The distribution functions of new clock times for events $e' \in N(s'; s, e)$ are as follows. If e' = "end of transmission" and $s' = (z_1, \dots, z_N; m; n)$, then the distribution function $F(x; s', s, e') = P\{L_m \leq x\}$. If e' = "observation of token" and $s' = (z_1, \dots, z_N; m; n)$, then the distribution function $F(x; s', s, e') = 1_{[R_{n-1}, \infty)}(x)$ if $n > 1$ and equals $1_{[R_n, \infty)}(x)$ if

$n = 1$. If $e^j =$ "arrival of packet for transmission by port j ," then the distribution function $F(x; s^j, e^j, s, e^j) = P\{A_j \leq x\}$.

As an application of Propositions (2.11) and (2.13), take $s_0^j = (0, 1, \dots, 1; 1; 2)$. Let $e^j =$ "observation of token" and $S^j = \{(z_1, \dots, z_N; 0; 1) \in S\}$ so that T_n is the n th time at which port 1 observes the token, $n \geq 0$. Observe that $T_n < \infty$ a.s. since

$$(3.3) \quad E\{T_n - T_{n-1}\} \leq R_1 + \dots + R_N + \sum_{j=1}^N E\{L_j\} < \infty$$

for all $n \geq 1$.

Let T_n^+ be the first time after T_{n-1} that the token leaves port N so that $S^+ = S^j$. For $s^+ = (z_1^+, \dots, z_N^+; m^+; n^+) \in S^+$, set $\mathcal{N}(s^+) = \{j: z_j^+(s^+) = 0\}$ and let $E^+(s^+) = \{e_j \in E: j \in \mathcal{N}(s^+)\}$ so that $E^+ = \{e_j \in E: j \leq N\}$. Take $R_{n,k}(s^+) = R_N$ for all $k = 1, 2, \dots, k(s^+)$ and $s^+ \in S^+$. Assume that the distribution of A_j is NBU and that

$$\delta_j = P\{A_j \leq R_N\} > 0$$

for $j = 1, 2, \dots, N$ so that

$$\delta(s^+) = \prod_{j \in \mathcal{N}(s^+)} \delta_j \geq \prod_{j=1}^N \delta_j = \delta > 0.$$

Then $P\{X(T_n) = s_0^j \text{ i.o.}\} = 1$.

A transition of the process $\{X(t): t \geq 0\}$ defined by Equation (3.2) to state s_0^j can occur when event e^j is the trigger event only if e^j occurs in state $s^j = (1, \dots, 1, 0, 1)$ and in this case the set $O(s_0^j; s^j, e^j) = \emptyset$. Since Equation (2.15) holds and $P\{X(T_n) = s_0^j \text{ i.o.}\} = 1$, the successive times T_n at which e^j triggers a transition (in state s^j) to state s_0^j are stopping times and regeneration points for the process $\{X(t): t \geq 0\}$. The expected time

between these regeneration points is finite by Equation (3.3). At these time points there is a packet queued for transmission at ports $2, 3, \dots, N$ and port 1 starts transmission of a packet.

(3.4) EXAMPLE (Collision-free bus network). Consider a bus network (Eswaran, Hamacher, and Shedler [5]) with N ports, numbered $1, 2, \dots, N$ from left to right; see Figure 2. Message packet traffic on the passive bilateral bus is transmitted/received by port j at tap $B(j)$. In addition to the bus, a one-way logic control wire also links the ports. Associated with each port j is a flip-flop, $S(j)$, called the *send flip-flop*. The signal $P(j)$, called the *OR-signal*, tapped at the control wire input to port j is the inclusive OR of the send flip-flops of all ports to the left of port j . Denote by T the end-to-end bus propagation delay. [For technical reasons, T actually must be the end-to-end propagation delay plus a small (fixed) quantity.] Denote the actual propagation delay along the bus between port i and port j by $T(i, j)$, $i, j = 1, 2, \dots, N$. Thus, $T(i, j) = T(j, i) < T$ for all i, j and $T(i, j) + T(j, k) = T(i, k)$ for all $i < j < k$. (We assume that $T(i, j) \neq T(k, j)$ for distinct i, k and all j .) Let $R(j)$ be the propagation delay (including gate delays) along the control wire from port j to port N , $j = 1, 2, \dots, N$; thus, $R(1) \geq R(2) \geq \dots \geq R(N) = 0$. Denote by $R(i, j)$ the propagation delay along the control wire from port i to port j . We assume that signal propagation along the control wire is slower than along the bus and that delays along shorter sections of each path scale proportionally; i.e., $R(1) > T$ and $R(i, j) > T(i, j)$ for all i, j .

Specification of distributed control scheme A_i is in terms of an algorithm for an individual port j . Packets (for transmission by port j) which arrive while an execution of the algorithm by port j is in progress queue externally. Upon completion of this

execution of the algorithm, one of any such packets immediately becomes available to port j for transmission and the next execution of the algorithm begins.

Algorithm A1

- Set $S(j)$ to 1
- Wait for a time interval $R(j) + T$
- Wait until the bus is observed (by port j) to be idle AND $P(j)=0$; then start transmission of the packet, simultaneously resetting $S(j)$ to 0.

For simplicity we assume that there can be at most one packet in queue at each port. Specifically, suppose that the time from end of transmission by port j until the arrival of a next packet for transmission by port j is a positive random variable, A_j , with finite mean. Also suppose that the time for port j to transmit a packet is a positive random variable, L_j , with finite mean and (so that Algorithm A2 of [5] guarantees transmission of all packets) such that $P\{L_j \leq R(1) + T\} = 0$.

Set

$$(3.5) \quad W(i) = (W_1(i), \dots, W_N(i)),$$

where $W_j(i)$ equals 1 if at time i port j has set its flip-flop but has not yet completed the $R(j) + T$ wait, equals 2 if port j has completed the $R(j) + T$ wait but has not started transmission, equals 3 if port j is transmitting, and equals 4 otherwise. Next set

$$(3.6) \quad U(i) = (U_1(i), \dots, U_N(i)),$$

where $U_j(i)$ equals k if port j observes transmission of a packet by port k on the bus at

time t , and equals 0 otherwise. Also set

$$(3.7) \quad V(t) = (V_{2,1}(t), V_{3,1}(t), V_{3,2}(t), V_{4,1}(t), \dots, V_{N,N-1}(t)),$$

where $V_{jk}(t)$ equals 1 if and only if $S(k)=1$ at time $t-R(k,j)$, and equals 0 otherwise.

(Port j observes $P(j)=1$ at time t if and only if $V_{jk}(t) = 1$ for some $k < j$.) Finally, set $Z(t) = 1$ if some port is transmitting at time t and this port started transmission when it observed an end of transmission; otherwise $Z(t) = 0$. Then set

$$(3.8) \quad X(t) = (W(t); Z(t); U(t); V(t)).$$

The stochastic process $\{X(t); t \geq 0\}$ defined by Equation (3.8) is a cmmp with a finite state space, S , and event set, E . The events in the set E are: "end of transmission by port j ," "end of wait for $R(j) + T$," "setting (to 1) of flip-flop by port j ," "observation by port j of start of transmission by port $k \neq j$," "observation by port j of end of transmission by port $k \neq j$," "observation by port j of end of transmission by port $k \neq j$ and start of transmission by port $l \neq j$," "observation by port j of the setting (to 1) of flip-flop by port k to the left," and "observation by port j of the resetting (to 0) of a flip-flop by port k to the left," $j = 1, 2, \dots, N$. For $s = (w_1, \dots, w_N; z; u_1, \dots, u_N; v_{2,1}, \dots, v_{N,N-1}) \in S$ the event sets $E(s)$ are as follows. The event set $E(s)$ contains "setting (to 1) of flip-flop by port j " if and only if $w_j = 4$. The event "end of transmission by port j " $\in E(s)$ if and only if $w_j = 3$. The event "end of wait for $R(j) + T$ " $\in E(s)$ if and only if $w_j = 1$. The event "observation by port j of start of transmission by port k " $\in E(s)$ if and only if (i) $w_k = 3$, $z = 0$, and $u_j \neq k$ or (ii) $w_k = 3$, $z = 1$ and either $u_j = 0$ or $u_j = l$ for some l between k and j . The event "observation by port j of end of transmission by port k " $\in E(s)$ if and only if $u_j = k$ and $w_k = 1$ or 4 and either $z = 0$ or $w_l \neq 3$ for all l between j and k . The event "observation by port j of end of transmission by port $k \neq j$ and start of transmission by port $l \neq j$ " $\in E(s)$ if and only if $u_j = k$, $z = 1$, and $w_l = 3$ with l between k and j . The event

"observation by port j of setting of flip-flop by port k to the left" $\in E(s)$ if and only if $w_k = 1$ and $v_{j,k} = 0$ for some $k < j$. The event "observation by port j of resetting of flip-flop by port k to the left" $\in E(s)$ if and only if $w_k = 3$ and $v_{j,k} = 1$ for some $k < j$.

Note that with this definition of the event sets $E(s)$ no "observation by port j of start of transmission by port k " and "observation by port j of end of transmission by port k " can occur simultaneously. To see this, let $k < l < j$. Suppose that port k ends transmission of a packet at time t and that port l starts transmission of a packet at time $t' = t + T(k, l)$. Then the event "observation by port j of end of transmission by port k and start of transmission by port l " is scheduled at time t' and (since $z = 1$ and $w_l = 3$ where l is between k and j) the event "observation by port j of end of transmission by port k " (which was scheduled at time t) ceases to be scheduled at time t' .

The distribution functions of new clock times for events $e' \in N(s'; s, e')$ are as follows. If $e' =$ "end of transmission by port j ," then the clock setting distribution function $F(x; s', e', s, e) = P\{L_j \leq x\}$. If $e' =$ "end of wait for $R(j) + T$," then the clock setting distribution function $F(x; s', e', s, e) = 1_{[R(j)+T, \infty)}(x)$. If $e' =$ "setting (to 1) of flip-flop by port j ," then the clock setting distribution function $F(x; s', e', s, e) = P\{A_j \leq x\}$. If $e' =$ "observation by port j of start of transmission by port k ," then the clock setting distribution function $F(x; s', e', s, e) = 1_{[T(k, j), \infty)}(x)$. If $e' =$ "observation by port j of end of transmission by port k ," then the clock setting distribution function $F(x; s', e', s, e) = 1_{[T(k, j), \infty)}(x)$. If $e' =$ "observation by port j of end of transmission by port k and start of transmission by port l ," then the clock setting distribution function $F(x; s', e', s, e) = 1_{[T(l, j), \infty)}(x)$. If $e' =$ "observation by port j of setting of flip-flop by port k to the left," then the clock setting distribution function $F(x; s', e', s, e) = 1_{[R(k, j), \infty)}(x)$.

If e' = "observation by port j of resetting of flip-flop by port k to the left," then the clock setting distribution function $F(x; s', e', s, e') = 1_{[R(k,j), \infty)}(x)$.

As an application of Propositions (2.11) and (2.13), take $s'_0 = (4, 2, \dots, 2; 0; 0, 1, \dots, 1; 0, 0, 1, \dots, 0, 1, \dots, 1)$. Let e' = "end of transmission by port 1" and $S' = \{(3, w_2, \dots, w_N; x; 1, \dots, 1; v) \in S : v_{j,1} = 0 \text{ for } j = 2, 3, \dots, N\}$ so that T_n is the n th time at which port 1 ends transmission, $n \geq 0$. Then port 1 ends transmission of a packet with each other port j having observed the resetting of port 1's flip-flop, having a packet queued for transmission, and having completed the $R(j) + T$ wait at time T_n if $X(T_n) = s'_0$. Observe that

$$(3.9) \quad T_n - T_{n-1} = A_{1n} + R(1) + T + D_n + L_{1n},$$

where L_{1n} is distributed as L_1 , A_{1n} is distributed as A_1 , and D_n is a nonnegative random variable. Provided that the distribution of L_j is NBU, it can be shown that

$$E\{D_n\} \leq \sum_{j=2}^N E\{L_j\}$$

so that

$$(3.10) \quad E\{T_n - T_{n-1}\} \leq E\{A_1\} + R(1) + T + \sum_{j=1}^N E\{L_j\} < \infty$$

and therefore $T_n < \infty$ a.s.

Let T_n^+ be the first time after T_{n-1} that port 1 begins transmission of a packet so that $S^+ = S'$. Let e_j = "setting of flip-flop by port j ." For $s^+ = (w^+, s^+, u^+, v^+) \in S^+$, set $\mathcal{K}(s^+) = \{j : u_j^+ = 4\}$ and let $E^+(s^+) = \{e_j \in E : j \in \mathcal{K}(s^+)\}$ so that $E^+ = \{e_2, \dots, e_N\}$. Take $R_{n,k}(s^+) = L_1 - (R(1) + T)$ for all $s^+ \in S^+$ and $k = 1, 2, \dots, k(s^+)$. Assume that the

distribution of A_j is NBU and that

$$\delta_j = P\{A_j + R(1) + T \leq L_1\} > 0,$$

$j = 2, 3, \dots, N$. It follows that

$$\delta = P\{A_j + R(1) + T \leq L_1, j = 2, 3, \dots, N\} > 0$$

so that

$$\delta(s^+) = P\{A_j + R(1) + T \leq L_1, j \in J(s^+)\} \geq \delta.$$

Then $P\{X(T_n) = s_0^+ \text{ i.o.}\} = 1$.

A transition of the process $\{X(t); t \geq 0\}$ defined by Equation (3.8) to state s_0^+ can occur when event e^+ is the trigger event only if e^+ occurs in state $s^+ = (3, 2, \dots, 2; 0; 1, \dots, 1; 0, 1, \dots, 1)$ and in this case the set $O(s_0^+; s^+, e^+) = \emptyset$. Since Equation (2.15) holds and $P\{X(T_n) = s_0^+ \text{ i.o.}\} = 1$, the successive times T_n at which e^+ triggers a transition (in state s^+) to state s_0^+ are stopping times and regeneration points for the process $\{X(t); t \geq 0\}$. The expected time between these regeneration points is finite by Equation (3.10).

(3.11) **EXAMPLE (Slotted ring).** Consider a ring network having a fixed number, K , of equal size *slots*, and a fixed number of equally spaced *ports*, labelled $1, 2, \dots, N$ in the direction of signal propagation; see Figure 3. At each port constant (slot size) length *message packets* arrive according to a random process and queue externally. The propagation delay from one port to the next is a positive constant, R . We assume that the number of ports, N , is a multiple of K and (so that there is no loss of utilization due to "unused bits") that the time to transmit a message packet is equal to NR/K . The lead "full/empty" (F/E) bit maintains the status of each slot. A port holds a slot from the

time that it begins filling the slot until it releases the slot. Subject to the restriction that no port can hold more than one slot simultaneously, a port that has a packet queued for transmission and observes the status bit of an empty slot sets the bit to 1 ("full") and starts transmission. Transmission ends when the slot contains the entire packet. When the status bit of the filled slot propagates back to the sending port, it resets the bit to 0 ("empty") and releases the slot. The port releases the slot even if it has another packet queued for transmission. This ensures that all ports have an opportunity to transmit. Assume that message packets arrive at individual ports randomly and independently of each other; i.e., the time from end of transmission by port j until the arrival of the next packet for transmission by port j is a positive random variable, A_j , with finite mean. Note that there is at most one packet queued for transmission at any time at any particular port.

Set

$$(3.12) \quad X(t) = (Z_1(t), \dots, Z_N(t); M_1(t), \dots, M_K(t); N_1(t), \dots, N_K(t)),$$

where

$$Z_j(t) = \begin{cases} 1 & \text{if there is a packet queued for transmission at port } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2, \dots, K$

$$M_i(t) = \begin{cases} j & \text{if port } j \text{ holds slot } i \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

$N_i(t) = j$ if at time t the status bit of slot i is propagating to port j , $j = 1, 2, \dots, N$. For any i ($1 \leq i \leq k$) the vector $(Z_1(t), \dots, Z_N(t); M_1(t), \dots, M_K(t); N_i(t))$ contains the same information as the vector $X(t)$. Incorporation of all the components $N_1(t), \dots, N_K(t)$ into the state

vector facilitates generation of the process.

The process $\{X(t); t \geq 0\}$ defined by Equation (3.12) is a GSMP with a finite state space, S , and event set, E . The events in the set E are: "observation of status bits by ports" and "arrival of packet for transmission by port j ," $j = 1, 2, \dots, N$. Let $s = (z_1, \dots, z_N; m_1, \dots, m_K; n_1, \dots, n_K) \in S$. The event "observation of status bits by ports" $\in E(s)$ for all $s \in S$. The event "arrival of message for transmission by port j " $\in E(s)$ if and only if $z_j = 0$ and for each i either (i) $m_i \neq j$ or (ii) $m_i = j$ and $n_i - 1 = j - 1 + l \pmod{N}$ for some integer l such that $N/K < l \leq N$. Note that the ends of transmission coincide with the occurrence of particular "observation of status bits by ports" events. Suppose, for example, that there are $N = 4$ ports and $K = 2$ slots and that $s = (0, 0, 0, 0; 1, 0; 3, 1)$; i.e., port 1 is transmitting a packet in slot 1, slot 2 is empty, the status bit of port 1 is propagating to port 3, and the status bit of slot 2 is propagating to port 1. Then the occurrence of the event "observation of status bits by ports" in state s corresponds to an end of transmission by port 1.

In a slotted ring with $N = 4$ ports and $K = 2$ slots, take $s'_0 = (0, 1, 0, 1; 1, 3; 2, 4)$. Let $e^* = \text{"observation of status bits by ports"}$ and $S^* = \{(z_1, z_2, z_3, z_4; 0, m_2; 1, 3) \in S\}$ so that T_n is the n th time at which port 1 observes the status bit of slot 1 and slot 1 is empty, $n \geq 0$. Suppose that the distribution of A_j is NBU and that $P\{A_j < R\} = 1$, $j = 1, 2, \dots, N$. Then $P\{X(T_n) = s'_0\} = 0$ for all $n \geq 1$ if $X(0) = (0, 1, 0, 1; 0, 3; 2, 4)$. Using arguments similar to those in the proof of Proposition (2.11) it can be shown that if the distribution, F_j , of A_j is NBU and $F_j(b) - F_j(a) > 0$ for all $0 \leq a < b < \infty$, then $P\{X(T_n) = s'_0 \text{ i.o.}\} = 1$.

4. CONCLUDING REMARKS

It is sometimes possible to establish recurrence results under weaker positivity assumptions than those required by hypothesis (ii) of Proposition (2.11). For example, in the token ring model of Example (3.1), $P\{X(T_n) = s_0 \text{ i.o.}\} = 1$ if the distribution of A_j is NBU and $P\{A_j \leq R_j + \dots + R_N\} > 0$.

APPENDIX

Let $\{X(t): t \geq 0\}$ be a GMP with finite state space, S , and event set, E . Recall that t_n is the time of the n th state transition and that $S_n = X(t_n)$ is the state of the system at time t_n , $n \geq 0$. Also recall that C_n is the vector of clock readings at time t_n and that $C_{n,i}$ is the i th coordinate of the vector C_n for $e_i \in E(S_n)$. Denote by $i_n^* = i^*(S_{n-1}, C_{n-1})$ the index of the n th trigger event and let $I_n = \{i: e_i \in E(S_n)\}$.

Let $s_0, s_1, \dots, s_n \in S$ and $e_{i_1}^*, \dots, e_{i_n}^* \in E$ with $p(s_k: s_{k-1}, e_{i_k}^*) > 0$. Then the joint event

$$(A.1) \quad \{X(t_n) = s_n, e_n^* = e_{i_n}^*, X(t_{n-1}) = s_{n-1}, e_n^* = e_{i_{n-1}}^*, \dots, e_1^* = e_{i_1}^*, X(0) = s_0\}$$

is equivalent to the joint event specified by the inequalities

$$(A.2) \quad C_{m, i_{m+1}^*} \leq C_{m, j}, \quad i \in I_m - \{i_{m+1}^*\} \text{ and } m = 0, 1, \dots, n-1$$

in conjunction with the equations

$$(A.3) \quad X(t_k) = S_k = s_k, \quad k = 0, 1, \dots, n.$$

If $I_m = \{i_{m+1}^*\}$, we write $C_{m, i_{m+1}^*} < \infty$.

We assume throughout that $E(s_0)$ is the set of active events at time $t = 0$ and that all active clocks are reset at time $t = 0$; i.e.,

$$P\{C_{0,i} \leq x\} = F(x; s', e_i, s, e)$$

for some $s, s' \in S$ and $e \in E$ (dependent on i), $e_i \in E(s_0)$. In addition, we define

$$N(s_m: s_{m-1}, e_m^*) = E(s_0) \text{ for } m = 0.$$

Next observe that if $e_i \in O(s_n; s_{n-1}, e_n^*)$ so that $C_{n,i}$ is an old clock reading, then

$$C_{n,i} = C_{m,i} - \sum_{k=m}^{n-1} C_{k,i_{k+1}},$$

where t_m is the latest time prior to t_n at which the clock associated with event e_i was set. This implies that any old clock reading $C_{k,i}$ appearing in Equation (A.2) can be expressed in terms of one or more $C_{m,j}$ with $e_j \in N(s_m; s_{m-1}, e_m^*)$ and $m \leq k$. Replacing in this manner all old clock readings appearing in Equation (A.2) by expressions which involve only new clock readings, we obtain an equivalent system of inequalities which, in conjunction with Equation (A.3), we denote by \mathfrak{W}_n . We call \mathfrak{W}_n the *canonical representation* of the joint event

$$\{X(t_n) = s_n, e_n^* = e_n^*, X(t_{n-1}) = s_{n-1}, e_{n-1}^* = e_{n-1}^*, \dots, e_1^* = e_1^*, X(0) = s_0\}.$$

(A.4) LEMMA. Let $1 \leq j_1 \leq j_2 \leq \dots \leq j_{l(n)} \leq n$ such that $N(s_{j_k}; s_{j_k-1}, e_{j_k}^*) \neq \emptyset$, $k = 1, 2, \dots, l(n)$.

Select $e_{i_k} \in N(s_{j_k}; s_{j_k-1}, e_{j_k}^*)$ and let

$$\mathfrak{V}_n = \{t_{j_k} + C_{j_k, i_k} > t_n, k = 1, 2, \dots, l(n)\}.$$

Either the set of inequalities $\{\mathfrak{W}_n, \mathfrak{V}_n\}$ is inconsistent or there exists $\mathfrak{W}_n \subset \mathfrak{W}_n$ such that (i) $\{\mathfrak{W}_n, \mathfrak{V}_n\}$ and $\{\mathfrak{W}_n, \mathfrak{V}_n\}$ are equivalent, (ii) no random variable C_{j_k, i_k} in \mathfrak{W}_n appears in \mathfrak{V}_n , and (iii) the random variables in \mathfrak{V}_n are mutually independent.

Proof: For fixed k , observe that the variable C_{j_k, i_k} appears only in those inequalities in \mathfrak{W}_n corresponding to state transitions at times $t_{j_k}, t_{j_k+1}, \dots, t_n$. There are two cases to consider.

Case (i). For some k and $j_k \leq l \leq n-1$, \mathfrak{W}_n contains the inequalities

$$\Psi\left(C_{j_k, j_k} - \sum_{m=j_k}^{l-1} C_{m, j_{m+1}}\right) < \Psi(C_{l, l}), \quad l \in I_1 - \{j_k\},$$

where $\Psi(\cdot)$ denotes an expression written in canonical form. By the structure of the case this means that

$$\Psi(C_{l, j_{l+1}}) = \Psi\left(C_{j_k, j_k} - \sum_{m=j_k}^{l-1} C_{m, j_{m+1}}\right)$$

which implies that

$$\Psi(C_{j_k, j_k} + \xi_k) = \Psi\left(\xi_{j_k} + \sum_{m=j_k}^l C_{m, j_{m+1}}\right) = \Psi(\xi_{l+1}) \leq \Psi(\xi_n)$$

and contradicts the corresponding inequality in \mathfrak{W}_n .

Case (ii). For every k ,

$$(A.5) \quad \Psi(C_{l, j_{l+1}}) < \Psi\left(C_{j_k, j_k} - \sum_{m=j_k}^{l-1} C_{m, j_{m+1}}\right), \quad l = j_k, j_{k+1}, \dots, n-1.$$

This is equivalent to

$$\Psi(C_{j_k, j_k} + \xi_{j_k}) > \Psi\left(\xi_{j_k} + \sum_{m=j_k}^l C_{m, j_{m+1}}\right) = \Psi(\xi_{l+1}), \quad l = j_k, j_{k+1}, \dots, n-1.$$

But clearly, (for every k) each of these equations is implied by the inequality

$$\Psi(C_{j_k, j_k} + \xi_{j_k}) > \Psi(\xi_n)$$

which is an element of \mathfrak{W}_n . Since the only inequalities in \mathfrak{W}_n which contain the random variable C_{j_k, j_k} are those in Equation (A.5), the required subset \mathfrak{W}_n is formed by deleting (for each k) the inequalities in Equation (A.5). Note that since the only $C_{l, j}$ variables

appearing in \mathcal{V}_n correspond to new clock settings, the construction of the ω_n ensures that the random variables in \mathcal{V}_n are mutually independent. \square

(4.6) LEMMA. Let $s^+ \in S^+$ and $x_0, \dots, x_{n-1} \in S$. Under the conditions of Proposition (2.11),

$$P\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+);$$

$$X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}$$

$$\geq \delta P\{X(T_n^+) = s^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}.$$

Proof: Set

$$U_n = \{X(T_n^+) = s_n^+, X(T_{n-1}) = x_{n-1}, \dots, X(T_0) = x_0\}$$

and let $\{V_n^j: j = 1, 2, \dots\}$ be the (countable) set of all joint events of the form

$$V_n^j = \{X(t_{j(1,n)}) = s_{j(1,n)}, e_{j(1,n)} = e_{j(1,n)}^*, \dots, e_1^* = e_1^*, X(0) = x_0\},$$

where $t_{j(1,n)} = T_n^+$ and there exist $l_1 < \dots < l_{n-1}$ such that $e_{l_i}^* = e^*$, $s_{l_i-1} \in S^+$, $s_{l_i} = x_{l_i}$, and either $e_i \neq e^*$ or $s_{i-1} \notin S^+$ for all $i \neq l_j$. Also let $\mathcal{V}_{j(1,n)}^j$ be the canonical representation of the joint event V_n^j . Next consider the joint event

$$\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+); U_n\}.$$

If $S_{n,k}(s^+) = T_n^+$, then the vacuous statement $\{A_{n,k}(s^+) > 0\}$ can be written as $\{S_{n,k}(s^+) + A_{n,k}(s^+) > T_n^+\}$. If $S_{n,k}(s^+) < T_n^+$, then $S_{n,k}(s^+) + A_{n,k}(s^+) > T_n^+$ since $A_{n,k}(s^+)$ is by definition the clock reading for an event that is active at time T_n^+ . Thus, the joint events

$$(4.7) \quad \{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+); U_n\}$$

and

$$(A.8) \quad \begin{aligned} &\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), \\ &\quad S_{n,k}(s^+) + A_{n,k}(s^+) > T_n^+, k = 1, 2, \dots, k(s^+); U_n\} \end{aligned}$$

are equivalent.

Now observe that for every sequence V_n^j of states and trigger events, $S_{n,k}(s^+)$ corresponds to some $\xi_{N(i,n,k)}$ and T_n^+ to some $\xi_{J(i,n)}$. Also, $A_{n,k}(s^+)$ corresponds to some $C_{N(i,n,k), m(i,n,k)}$, where $e_{m(i,n,k)} \in N(s_{N(i,n,k)}; s_{N(i,n,k)-1}, e_{N(i,n,k)}^*)$, and $R_{n,k}(s^+)$ corresponds to some $R_{q(i,n),k}(s^+)$. Since U_n is the disjoint union of the events V_n^j , we can combine the above results to obtain

$$(A.9) \quad \begin{aligned} &P\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+); U_n\} \\ &= \sum_i P\{C_{N(i,n,k), m(i,n,k)} \leq \xi_{J(i,n)} - \xi_{N(i,n,k)} + R_{q(i,n),k}(s^+), \\ &\quad C_{N(i,n,k), m(i,n,k)} > \xi_{J(i,n)} - \xi_{N(i,n,k)}, k = 1, 2, \dots, k(s^+); \mathfrak{V}_{J(i,n)}^j\}, \end{aligned}$$

where all terms of probability zero are excluded from the sum. By Lemma (A.4), we can replace $\mathfrak{V}_{J(i,n)}^j$ by $\mathfrak{V}_{J(i,n)}$ without altering the value of the sum.

Setting

$$Z_{J(i,n), N(i,n,k)} = \mathfrak{V}(\xi_{J(i,n)} - \xi_{N(i,n,k)}) = \mathfrak{V}\left(\sum_{j=0}^{N(i,n,k)-1} C_{J, j+1}^* - \sum_{j=0}^{N(i,n,k)-1} C_{J, j+1}^*\right),$$

and denoting the set of random variables appearing in the canonical representation

$\mathfrak{V}(\xi_{J(i,n)} - \xi_{N(i,n,k)})$ by $\mathcal{V}(\xi_{J(i,n)} - \xi_{N(i,n,k)})$, we can write

$$P\{C_{N(i,n),m(i,n,k)} \leq Z_{N(i,n),N(i,n,k)} + R_{q(i,n),k}(s^+),$$

$$C_{N(i,n),m(i,n,k)} > Z_{N(i,n),N(i,n,k)}, k = 1, 2, \dots, k(s^+); \mathfrak{U}_{N(i,n)}^i\}.$$

$$(A.10) \quad = \int \int P\{C_{N(i,n),m(i,n,k)} \leq Z_{N(i,n),N(i,n,k)}(c) + r_{q(i,n),k},$$

$$C_{N(i,n),m(i,n,k)} > Z_{N(i,n),N(i,n,k)}(c), k = 1, 2, \dots, k(s^+); \mathfrak{U}_{N(i,n)}^i(c)\} dF_R(r) dF_C(c),$$

where F_R and F_C are the joint distribution functions of $\{R_{q(i,n),k}(s^+), k = 1, 2, \dots, k(s^+)\}$ and $\{Z_{N(i,n),N(i,n,k)}, k = 1, 2, \dots, k(s^+)\}$, respectively. Using hypothesis (II) of Proposition (2.11) and the fact that the random variables $\{C_{N(i,n),m(i,n,k)}, k = 1, 2, \dots, k(s^+)\}$ do not appear in $\mathfrak{U}_{N(i,n)}^i$, we have that

$$P\{C_{N(i,n),m(i,n,k)} \leq Z_{N(i,n),N(i,n,k)}(c) + r_{q(i,n),k},$$

$$C_{N(i,n),m(i,n,k)} > Z_{N(i,n),N(i,n,k)}(c), k = 1, 2, \dots, k(s^+); \mathfrak{U}_{N(i,n)}^i(c)\}$$

$$= \prod_{k=1}^{k(s^+)} [P\{C_{N(i,n),m(i,n,k)} \leq Z_{N(i,n),N(i,n,k)}(c) + r_{q(i,n),k},$$

$$C_{N(i,n),m(i,n,k)} > Z_{N(i,n),N(i,n,k)}(c)\}] P\{\mathfrak{U}_{N(i,n)}^i(c)\}$$

$$\geq \prod_{k=1}^{k(s^+)} [P\{C_{N(i,n),m(i,n,k)} \leq r_{q(i,n),k}\}]$$

$$P\{C_{N(i,n),m(i,n,k)} > Z_{N(i,n),N(i,n,k)}(c)\}] P\{\mathfrak{U}_{N(i,n)}^i(c)\}$$

$$= P\{C_{N(i,n),m(i,n,k)} \leq r_{q(i,n),k}, k = 1, 2, \dots, k(s^+)\}$$

$$P\{C_{N(i,n),m(i,n,k)} > Z_{N(i,n),N(i,n,k)}, k = 1, 2, \dots, k(s^+); \mathfrak{U}_{N(i,n)}^i(c)\}.$$

Using hypothesis (III), substituting the right hand side into Equation (A.10) and integrating yields

$$\begin{aligned}
 & P\{C_{N(i,n),m(i,n,k)} \leq \xi_{f(i,n)} - \xi_{N(i,n,k)} + R_{q(i,n),k}, \\
 (A.11) \quad & C_{N(i,n),m(i,n,k)} > \xi_{f(i,n)} - \xi_{N(i,n,k)}, k = 1, 2, \dots, k(s^+); \overline{U}_{f(i,n)}^i\} \\
 & \geq \delta P\{C_{N(i,n),m(i,n,k)} > \xi_{f(i,n)} - \xi_{N(i,n,k)}, k = 1, 2, \dots, k(s^+); \overline{U}_{f(i,n)}^i\}.
 \end{aligned}$$

Substituting Equation (A.11) into Equation (A.9) and using Lemma (A.4),

$$\begin{aligned}
 & P\{S_{n,k}(s^+) + A_{n,k}(s^+) \leq T_n^+ + R_{n,k}(s^+), k = 1, 2, \dots, k(s^+); U_n\} \\
 & \geq \sum_i \delta P\{C_{N(i,n),m(i,n,k)} > \xi_{f(i,n)} - \xi_{N(i,n,k)}, k = 1, 2, \dots, k(s^+); \overline{U}_n^i\} \\
 & = \delta P\{S_{n,k}(s^+) + A_{n,k}(s^+) > T_n^+, k = 1, 2, \dots, k(s^+); U_n\} \\
 & = \delta P\{U_n\}.
 \end{aligned}$$

The last equality follows by the same reasoning that leads to the equivalence of the events in Equations (A.7) and (A.8). \square

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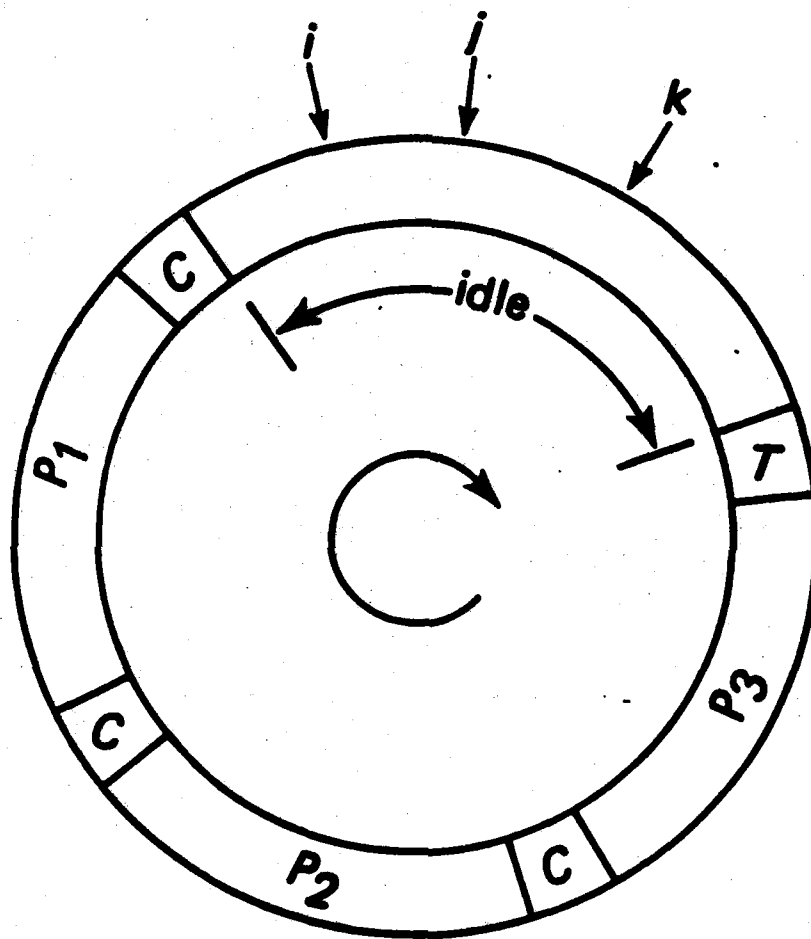


Figure 1. Token ring.



Figure 2. Collision-free bus network.

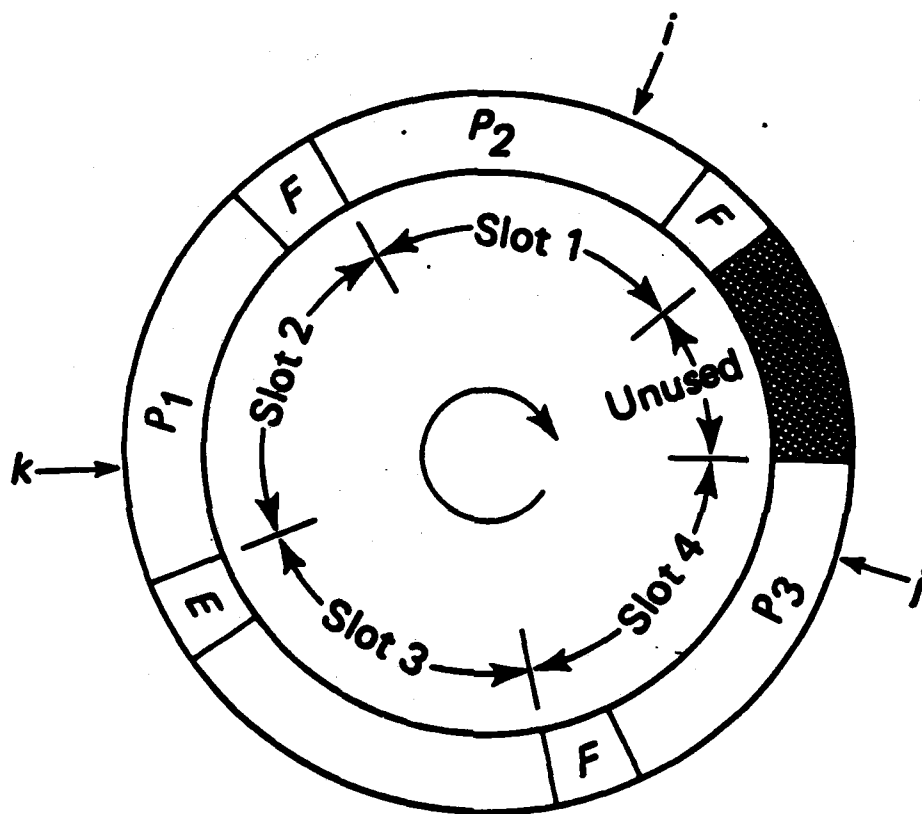


Figure 3. Slotted ring.

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